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Abstract: The aim of this paper is to introduce the notion of contra-generalized c^* -irresolute functions in topological spaces and study their basic properties. Also, we see that composition of two contra-generalized c^* -irresolute functions is contra-generalized c^* -irresolute function. This is the main part of this paper. Also, the contra-generalized c^* -irresolute function of a generalized c^* -irresolute function is contra-generalized c^* -irresolute function. Further, we prove contra-generalized c^* -irresolute function is the stronger form of contra- gc^* -continuous function.

Key words: gc^* -continuous functions, contra- gc^* -continuous functions, gc^* -irresolute functions and contra- gc^* -irresolute functions.

2010 Mathematics Subject Classification: 54A05, 54C99

1. Introduction

In 1963, Norman Levine [5] introduced semi-open sets in topological spaces. Also in 1970, he introduced the concept of generalized closed sets. In 1980, Jain [4] introduced totally continuous functions. Dontchev [3] introduced the notions of contra continuity in topological spaces in 1996. In 2011, S.S. Benchalli and Umadevi I Neeli [1] introduced the concept of semi-totally continuous functions in topological spaces. In this paper we introduce contra-generalized c^* -irresolute functions in topological spaces and study their basic properties. Section 2 deals with the preliminary concepts. In section 3, contra-generalized c^* -irresolute functions are introduced and study their basic properties.

2. Preliminaries

Throughout this paper X denotes a topological space on which no separation axiom is assumed. For any subset A of X , $cl(A)$ denotes the closure of A , $int(A)$ denotes the interior of A . Further $X \setminus A$ denotes the complement of A in X . The following definitions are very useful in the subsequent sections.

Definition: 2.1 [5] A subset A of a topological space X is said to be a semi-open set if $A \subseteq cl(int(A))$ and a semi-closed set if $int(cl(A)) \subseteq A$.

Definition: 2.2 [14] A subset A of a topological space X is said to be a α -open set if $A \subseteq int(cl(int(A)))$ and a α -closed set if $cl(int(cl(A))) \subseteq A$.

Definition: 2.3 [6] A subset A of a topological space X is said to be a c^* -open set if $int(cl(A)) \subseteq A \subseteq cl(int(A))$.

Definition: 2.4 [6] A subset A of a topological space X is said to be a generalized c^* -closed set (briefly, gc^* -closed set) if $cl(A) \subseteq H$ whenever $A \subseteq H$ and H is c^* -open. The complement of the gc^* -closed set is gc^* -open [7].

Definition: 2.5 [9] A subset A of a topological space X is said to be a pre-generalized c^* -closed set (briefly, pgc^* -closed set) if $pcl(A) \subseteq H$ whenever $A \subseteq H$ and H is c^* -open. The complement of the pgc^* -closed set is pgc^* -open [10].

Definition: 2.6 A function $f : X \rightarrow Y$ is called

- i. totally-continuous [4] if the inverse image of every open subset of Y is clopen in X ,
- ii. strongly-continuous [15] if the inverse image of every subset of Y is clopen subset of X ,
- iii. semi-totally continuous [1] if the inverse image of every semi-open subset of Y is clopen in X

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iv. contra-continuous [3] if the inverse image of every open subset of Y is closed in X .

Definition: 2.7 [8] A function $f : X \rightarrow Y$ is called a generalized c^* -continuous (briefly, gc^* -continuous) function if the inverse image of every closed subset of Y is gc^* -closed in X .

Definition: 2.8 [11] A function $f : X \rightarrow Y$ is called a pre-generalized c^* -continuous (briefly, pgc^* -continuous) function if the inverse image of every closed subset of Y is pgc^* -closed in X .

Definition: 2.9 [12] Let X and Y be two topological spaces. A function $f : X \rightarrow Y$ is called a contra-generalized c^* -continuous (briefly, contra- gc^* -continuous) function if $f^{-1}(V)$ is gc^* -closed in X for every open set V of Y .

Definition: 2.10 [13] Let X and Y be two topological spaces. A function $f : X \rightarrow Y$ is called a contra-pre-generalized c^* -continuous (briefly, contra- pgc^* -continuous) function if $f^{-1}(V)$ is pgc^* -closed in X for every open set V of Y .

Definition: 2.11 [2] A function $f : X \rightarrow Y$ is called an irresolute function if the inverse image of every semi-open subset of Y is semi-open in X .

Definition: 2.12 [8] A function $f : X \rightarrow Y$ is called a generalized c^* -irresolute (briefly, gc^* -irresolute) function if the inverse image of every gc^* -closed subset of Y is gc^* -closed in X .

Definition: 2.13 [11] A function $f : X \rightarrow Y$ is called a pre-generalized c^* -irresolute (briefly, pgc^* -irresolute) function if the inverse image of every pgc^* -closed subset of Y is pgc^* -closed in X .

3. Contra-generalized c^* -irresolute functions

In this section, we introduce contra-generalized c^* -irresolute functions and study their basic properties. Now, we begin with the definition of contra-generalized c^* -irresolute function.

Definition: 3.1 Let X and Y be two topological spaces. A function $f : X \rightarrow Y$ is said to be a contra-generalized c^* -irresolute (briefly, contra- gc^* -irresolute) function if $f^{-1}(V)$ is gc^* -closed in X for every gc^* -open set V of Y .

Example: 3.2 Let $X = \{1, 2, 3, 4\}$ and $Y = \{a, b, c, d\}$. Then, clearly $\tau = \{\emptyset, \{1\}, \{2, 3, 4\}, X\}$ is a topology on X and $\sigma = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}, Y\}$ is a topology on Y . Define f by $f(1) = b$, $f(2) = d$, $f(3) = c$, $f(4) = a$. Then the inverse image of every gc^* -open set in Y is gc^* -closed in X . Therefore, f is contra- gc^* -irresolute.

Proposition: 3.3 Let X, Y be two topological spaces. Then $f : X \rightarrow Y$ is contra- gc^* -irresolute iff $f^{-1}(U)$ is gc^* -open in X for every gc^* -closed set U of Y .

Proof: Assume that $f : X \rightarrow Y$ is contra- gc^* -irresolute. Let U be a gc^* -closed set in Y . Then $Y \setminus U$ is a gc^* -open set in Y . This implies, $f^{-1}(Y \setminus U)$ is a gc^* -closed set in X . Since $f^{-1}(Y \setminus U) = X \setminus f^{-1}(U)$, we have $X \setminus f^{-1}(U)$ is a gc^* -closed set in X . This implies, $f^{-1}(U)$ is a gc^* -open set in X . Conversely, assume that $f^{-1}(U)$ is gc^* -open in X for every gc^* -closed set U in Y . Let V be a gc^* -open set in Y . Then $Y \setminus V$ is gc^* -closed in Y . Therefore, $f^{-1}(Y \setminus V)$ is gc^* -open in X . That is, $X \setminus f^{-1}(V)$ is gc^* -open in X . This implies, $f^{-1}(V)$ is gc^* -closed in X . Therefore, f is contra- gc^* -irresolute.

Proposition: 3.4 Let X, Y be two topological spaces. Then every contra- gc^* -irresolute function is contra- gc^* -continuous.

Proof: The proof follows easily from the fact that every closed set is gc^* -closed.

The following example shows that the converse of Proposition 3.4 need not be true.

Example: 3.5 Let $X = \{a, b, c, d\}$ with topology $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}, X\}$ and $Y = \{1, 2, 3, 4\}$ with topology $\sigma = \{\emptyset, \{1\}, \{3\}, \{1, 3\}, \{2, 3, 4\}, Y\}$. Define $f : X \rightarrow Y$ by $f(a) = 4$, $f(b) = 2$, $f(c) = 1$, $f(d) = 3$. Then the inverse image of every open set in Y is gc^* -closed in X . Therefore, f is contra- gc^* -continuous. But f is not a contra- gc^* -irresolute, since the inverse image of the gc^* -open set $\{2\}$ is $\{b\}$, which is not a gc^* -closed set in X .

Proposition: 3.6 Let X, Y be two topological spaces. Then every contra- gc^* -irresolute function is contra- pgc^* -continuous.

Proof: Let $f : X \rightarrow Y$ be a contra-gc*-irresolute function. Let V be a closed set in Y . Then V is gc*-closed in Y . By our assumption, $f^{-1}(V)$ is gc*-open in X . This implies, $f^{-1}(V)$ is pgc*-open in X . Therefore, f is contra-pgc*-continuous.

The converse of Proposition 3.6 need not be true as seen from the following example.

Example: 3.7 Let $X = \{a, b, c, d\}$ with topology $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, b, c\}, X\}$ and $Y = \{1, 2, 3, 4, 5\}$ with topology $\sigma = \{\emptyset, \{1, 2\}, \{3, 4\}, \{1, 2, 3, 4\}, Y\}$. Define $f: X \rightarrow Y$ by $f(a) = 2, f(b) = 1, f(c) = 4, f(d) = 3$. Then the inverse image of every open set in Y is pgc*-closed in X . Therefore, f is contra-pgc*-continuous. But f is not a contra-gc*-irresolute, since the inverse image of the gc*-open set $\{1\}$ in Y is $\{b\}$, which is not a gc*-closed set in X .

Proposition: 3.8 Let X, Y and Z be topological spaces. Then

1. If $f : X \rightarrow Y$ is contra-gc*-irresolute and $g : Y \rightarrow Z$ is continuous (resp. gc*-continuous), then $g \circ f : X \rightarrow Z$ is contra-gc*-continuous,
2. If $f : X \rightarrow Y$ is contra-gc*-irresolute and $g : Y \rightarrow Z$ is gc*-irresolute, then $g \circ f : X \rightarrow Z$ is contra-gc*-irresolute,
3. If $f : X \rightarrow Y$ is contra-gc*-irresolute and $g : Y \rightarrow Z$ is gc*-irresolute, then $g \circ f : X \rightarrow Z$ is contra-gc*-continuous &
4. If $f : X \rightarrow Y$ is gc*-irresolute and $g : Y \rightarrow Z$ is contra-gc*-irresolute, then $g \circ f : X \rightarrow Z$ is contra-gc*-irresolute.

Proof: It is obvious.

The following Proposition shows that the composition of contra-gc*-irresolute function and some functions is contra-pgc*-continuous function.

Proposition: 3.9 Let X, Y and Z be topological spaces. Then

1. If $f : X \rightarrow Y$ is contra-gc*-irresolute and $g : Y \rightarrow Z$ is continuous, then $g \circ f : X \rightarrow Z$ is contra-pgc*-continuous,
2. If $f : X \rightarrow Y$ is contra-gc*-irresolute and $g : Y \rightarrow Z$ is gc*-continuous, then $g \circ f : X \rightarrow Z$ is contra-pgc*-continuous,
3. If $f : X \rightarrow Y$ is pgc*-irresolute and $g : Y \rightarrow Z$ is contra-gc*-irresolute, then $g \circ f : X \rightarrow Z$ is contra-pgc*-continuous.

Proposition: 3.10 Let X, Y and Z be topological spaces.

1. If $f : X \rightarrow Y$ is contra-gc*-irresolute and $g : Y \rightarrow Z$ is contra-continuous (resp. contra-gc*-continuous), then $g \circ f : X \rightarrow Z$ is gc*-continuous,
2. If $f : X \rightarrow Y$ is contra-gc*-irresolute and $g : Y \rightarrow Z$ is contra-gc*-continuous, then $g \circ f : X \rightarrow Z$ is gc*-continuous (resp. pgc*-continuous),
3. If $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ are contra-gc*-irresolute, then $g \circ f : X \rightarrow Z$ is gc*-continuous (resp. pgc*-continuous) &
4. If $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ are contra-gc*-irresolute, then $g \circ f : X \rightarrow Z$ is gc*-irresolute.

Proof: The proof is trivial.

Proposition: 3.11 Let X, Y and Z be topological spaces. Then

1. If $f : X \rightarrow Y$ is contra-gc*-irresolute and $g : Y \rightarrow Z$ is totally-continuous, then $g \circ f : X \rightarrow Z$ is gc*-continuous (resp. pgc*-continuous),
2. If $f : X \rightarrow Y$ is contra-gc*-irresolute and $g : Y \rightarrow Z$ is strongly-continuous, then $g \circ f : X \rightarrow Z$ is gc*-continuous (resp. pgc*-continuous),
3. If $f : X \rightarrow Y$ is contra-gc*-irresolute and $g : Y \rightarrow Z$ is semi-totally continuous, then $g \circ f : X \rightarrow Z$ is gc*-continuous (resp. pgc*-continuous).

Proposition: 3.12 Let X, Y and Z be topological spaces. Then

1. If $f : X \rightarrow Y$ is contra-gc*-irresolute and $g : Y \rightarrow Z$ is totally-continuous, then $g \circ f : X \rightarrow Z$ is contra-gc*-continuous,

2. If $f: X \rightarrow Y$ is contra-gc*-irresolute and $g: Y \rightarrow Z$ is strongly-continuous, then $g \circ f: X \rightarrow Z$ is contra-gc*-continuous,
3. If $f: X \rightarrow Y$ is contra-gc*-irresolute and $g: Y \rightarrow Z$ is semi-totally continuous, then $g \circ f: X \rightarrow Z$ is contra-gc*-continuous.

In other words, totally-continuous (resp. strongly-continuous, semi-totally continuous) function of a contra-gc*-irresolute function is contra-gc*-continuous.

Proposition: 3.13 Let X, Y and Z be topological spaces. Then

1. If $f: X \rightarrow Y$ is contra-gc*-irresolute and $g: Y \rightarrow Z$ is totally-continuous, then $g \circ f: X \rightarrow Z$ is contra-pgc*-continuous,
2. If $f: X \rightarrow Y$ is contra-gc*-irresolute and $g: Y \rightarrow Z$ is strongly-continuous, then $g \circ f: X \rightarrow Z$ is contra-pgc*-continuous&
3. If $f: X \rightarrow Y$ is contra-gc*-irresolute and $g: Y \rightarrow Z$ is semi-totally continuous, then $g \circ f: X \rightarrow Z$ is contra-pgc*-continuous.

Proof: The proof follows from the fact that every gc*-closed set is pgc*-closed.

Irresolute and contra-gc*-irresolute functions are independent of each other. For example, let $X=\{1,2,3,4\}$ and $Y=\{a,b,c,d\}$. Then, clearly $\tau=\{\phi, \{1\}, \{3\}, \{4\}, \{1,3\}, \{1,4\}, \{3,4\}, \{1,3,4\}, X\}$ is a topology on X and $\sigma=\{\phi, \{a\}, \{b\}, \{a,b\}, \{b,c\}, \{a,b,c\}, \{a,b,d\}, Y\}$ is a topology on Y . Define $f: X \rightarrow Y$ by $f(1)=b, f(2)=d, f(3)=c, f(4)=a$. Then f is irresolute. But f is not contra-gc*-irresolute, since the inverse image of the gc*-open set $\{a,c\}$ is $\{3,4\}$, which is not a gc*-closed set in X . Define $g: X \rightarrow Y$ by $g(1)=g(3)=g(4)=a, g(2)=b$. Then g is contra-gc*-irresolute. But the inverse image of the semi-open set $\{b,c,d\}$ is $\{2\}$, which is not a semi-open set in X . Therefore, g is not irresolute.

Proposition: 3.14 Let X, Y and Z be topological spaces. If $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ are contra-gc*-irresolute, then $g \circ f: X \rightarrow Z$ is contra-gc*-irresolute.

Proof: Let $X=\{x_1, x_2, \dots, x_n\}$, $Y=\{y_1, y_2, \dots, y_m\}$ and $Z=\{z_1, z_2, \dots, z_p\}$ with $n < m < p$. Then there are four cases arise.

Case (i) : Suppose f and g are 1-1. Define $f \& g$ by $f(x_i)=y_i$ for all $i=1, 2, \dots, n$ and $g(y_j)=z_j$ for all $j=1, 2, \dots, m$. Since each $\{y_i\}$ is gc*-open and f is contra-gc*-irresolute, we have $\{f^{-1}(y_i)\}=\{x_i\}$ is gc*-closed. That is, each $\{x_i\}$ is gc*-closed in X . Since union of gc*-closed sets is gc*-closed, we have every subset of X is gc*-closed in X . This implies, inverse image of every gc*-open set in Z under $g \circ f$ is gc*-closed in X . Therefore, $g \circ f$ is contra-gc*-irresolute.

Case (ii) : Suppose f is 1-1 and g is not 1-1. Define f by $f(x_1)=y_1, f(x_2)=y_2, \dots, f(x_n)=y_n$ and define g by $g(y_1)=g(y_2)=\dots=g(y_k)=z_k, g(y_{k+1})=z_{k+1}, \dots, g(y_m)=z_m$, where $1 < k < m$. Since every singleton sets are gc*-open, we have $\{z_k\}, \{z_{k+1}\}, \dots, \{z_m\}$ are gc*-open in Z & $\{y_1\}, \{y_2\}, \dots, \{y_n\}$ are gc*-open in Y . Also, since f and g are contra-gc*-irresolute, we have $g^{-1}(\{z_k\}), g^{-1}(\{z_{k+1}\}), \dots, g^{-1}(\{z_m\})$ are gc*-closed in Y and $f^{-1}(\{y_1\}), f^{-1}(\{y_2\}), \dots, f^{-1}(\{y_n\})$ are gc*-closed in X . That is, $\{y_1, y_2, \dots, y_k\}, \{y_{k+1}, \dots, y_m\}$ are gc*-closed in Y and $\{x_1, x_2, \dots, x_n\}$ are gc*-closed in X . Since union of gc*-closed sets in X is gc*-closed in X , we have every subset of X is gc*-closed in X . This implies, inverse image of every gc*-open set in Z under $g \circ f$ is gc*-closed in X . Therefore, $g \circ f$ is contra-gc*-irresolute.

Case (iii) : Suppose g is 1-1 and f is not 1-1. Define f by $f(x_1)=f(x_2)=\dots=f(x_k)=y_k, f(x_{k+1})=y_{k+1}, \dots, f(x_n)=y_n$, where $1 < k < n$ and define g by $g(y_1)=z_1, g(y_2)=z_2, \dots, g(y_m)=z_m$. Since each $\{z_i\}$ and each $\{y_i\}$ are gc*-open in Z and Y respectively and $f \& g$ are contra-gc*-irresolute, we have each $\{y_i\}$ is gc*-closed in Y and $\{x_1, x_2, \dots, x_k\}, \{x_{k+1}\}, \dots, \{x_n\}$ are gc*-closed in X . Now,

$$(g \circ f)^{-1}(\{z_1\})=f^{-1}(g^{-1}(\{z_1\}))=f^{-1}(\{y_1\})=\phi,$$

$$(g \circ f)^{-1}(\{z_2\})=f^{-1}(g^{-1}(\{z_2\}))=f^{-1}(\{y_2\})=\phi,$$

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$$(g \circ f)^{-1}(\{z_{k-1}\}) = f^{-1}(g^{-1}(\{z_{k-1}\})) = f^{-1}(\{y_{k-1}\}) = \phi, \\ (g \circ f)^{-1}(\{z_k\}) = f^{-1}(g^{-1}(\{z_k\})) = f^{-1}(\{y_k\}) = \{x_1, x_2, \dots, x_k\}, \\ (g \circ f)^{-1}(\{z_{k+1}\}) = f^{-1}(g^{-1}(\{z_{k+1}\})) = f^{-1}(\{y_{k+1}\}) = \{x_{k+1}\},$$

$$(g \circ f)^{-1}(\{z_n\}) = f^{-1}(g^{-1}(\{z_n\})) = f^{-1}(\{y_n\}) = \{x_n\}, \\ (g \circ f)^{-1}(\{z_{n+1}\}) = f^{-1}(g^{-1}(\{z_{n+1}\})) = f^{-1}(\{y_{n+1}\}) = \phi,$$

$$(g \circ f)^{-1}(\{z_m\}) = f^{-1}(g^{-1}(\{z_m\})) = f^{-1}(\{y_m\}) = \phi, \\ (g \circ f)^{-1}(\{z_{m+1}\}) = f^{-1}(g^{-1}(\{z_{m+1}\})) = f^{-1}(\{\phi\}) = \phi,$$

$$(g \circ f)^{-1}(\{z_p\}) = f^{-1}(g^{-1}(\{z_p\})) = f^{-1}(\{\phi\}) = \phi, \text{ which are } gc^*\text{-closed in } X.$$

That is, each $(g \circ f)^{-1}(\{z_i\})$ is gc^* -closed in X .

If $\{z_1, z_2, \dots, z_s\}$, $1 < s < p$ is gc^* -open in Z , then $(g \circ f)^{-1}(\{z_1, z_2, \dots, z_s\}) = (g \circ f)^{-1}(\{z_1\}) \cup (g \circ f)^{-1}(\{z_2\}) \cup \dots \cup (g \circ f)^{-1}(\{z_s\})$.

Since the union of gc^* -closed sets in gc^* -closed, we have $(g \circ f)^{-1}(\{z_1, z_2, \dots, z_s\})$ is gc^* -closed. Therefore, inverse image of every gc^* -open set in Z under $g \circ f$ is gc^* -closed in X . Hence $g \circ f$ is contra- gc^* -irresolute.

Case (iv) : Suppose f and g are not 1-1. Define f by $f(x_1) = f(x_2) = \dots = f(x_k) = y_k$, $f(x_{k+1}) = y_{k+1}, \dots, f(x_n) = y_n$, where $1 < k < n$. Define g by $g(y_1) = g(y_2) = \dots = g(y_r) = z_r$, $g(y_{r+1}) = z_{r+1}, \dots, g(y_m) = z_m$, where $1 < r < m$. Since each $\{z_i\}$ and each $\{y_i\}$ are gc^* -open in Z and Y respectively and f, g are contra- gc^* -irresolute, we have $\{x_1, x_2, \dots, x_k\}, \{x_{k+1}\}, \dots, \{x_n\}$ are gc^* -closed in X and $\{y_1, y_2, \dots, y_r\}, \{y_{r+1}\}, \dots, \{y_m\}$ are gc^* -closed in Y . Now,

$$(g \circ f)^{-1}(\{z_1\}) = f^{-1}(g^{-1}(\{z_1\})) = f^{-1}(\{\phi\}) = \phi,$$

$$(g \circ f)^{-1}(\{z_{r-1}\}) = f^{-1}(g^{-1}(\{z_{r-1}\})) = f^{-1}(\{\phi\}) = \phi, \\ (g \circ f)^{-1}(\{z_r\}) = f^{-1}(g^{-1}(\{z_r\})) = f^{-1}(\{y_1, y_2, \dots, y_r\}) \\ = f^{-1}(\{y_1\}) \cup f^{-1}(\{y_2\}) \cup \dots \cup f^{-1}(\{y_r\}) \\ = \phi \cup \phi \cup \dots \cup f^{-1}(\{y_r\}) \\ = f^{-1}(\{y_r\}) \\ = \begin{cases} \{x_1, x_2, \dots, x_k\} \cup \{x_{k+1}\} \cup \dots \cup \{x_r\} \cup \{x_{r+1}\} & \text{if } r < k \\ \{x_1, x_2, \dots, x_{r+1}\} & \text{if } r \geq k \end{cases} \\ = \{x_1, x_2, \dots, x_{r+1}\} \\ (g \circ f)^{-1}(\{z_{r+1}\}) = f^{-1}(g^{-1}(\{z_{r+1}\})) = f^{-1}(\{y_{r+1}\}) \\ = f^{-1}(\{y_1, y_2, \dots, y_r\}) \cup \{x_{k+1}\} \cup \dots \cup \{x_r\} \cup \{x_{r+1}\} \\ = \begin{cases} \{x_1, x_2, \dots, x_k\} \cup \{x_{k+1}\} \cup \dots \cup \{x_r\} \cup \{x_{r+1}\} & \text{if } r+1 < k \\ \{x_1, x_2, \dots, x_{r-1}\} & \text{if } r+1 \geq k \end{cases} \\ = \{x_1, x_2, \dots, x_{r-1}\} \quad \text{if } r \geq k-1$$

$$(g \circ f)^{-1}(\{z_n\}) = f^{-1}(g^{-1}(\{z_n\})) = f^{-1}(\{y_n\}) = \{x_n\},$$

$$(g \circ f)^{-1}(\{z_{n+1}\}) = f^{-1}(g^{-1}(\{z_{n+1}\})) = f^{-1}(\{y_{n+1}\}) = \emptyset,$$

$$(g \circ f)^{-1}(\{z_m\}) = f^{-1}(g^{-1}(\{z_m\})) = f^{-1}(\{y_m\}) = \emptyset,$$

$$(g \circ f)^{-1}(\{z_{m+1}\}) = f^{-1}(g^{-1}(\{z_{m+1}\})) = f^{-1}(\{\emptyset\}) = \emptyset,$$

$$(g \circ f)^{-1}(\{z_p\}) = f^{-1}(g^{-1}(\{z_p\})) = f^{-1}(\{\emptyset\}) = \emptyset, \text{ which are } g\text{-closed in } X.$$

That is, each $(g \circ f)^{-1}(\{z_i\})$ is $g\text{-closed in } X$.

$$\text{If } \{z_1, z_2, \dots, z_q\}, \quad 1 < q < p \quad \text{is } g\text{-open in } Z, \quad \text{then } (g \circ f)^{-1}(\{z_1, z_2, \dots, z_q\}) = (g \circ f)^{-1}(\{z_1\}) \cup (g \circ f)^{-1}(\{z_2\}) \cup \dots \cup (g \circ f)^{-1}(\{z_q\}).$$

Since the union of $g\text{-closed sets}$ in $g\text{-closed}$, we have $(g \circ f)^{-1}(\{z_1, z_2, \dots, z_s\})$ is $g\text{-closed}$. Therefore, inverse image of every $g\text{-open set}$ in Z under $g \circ f$ is $g\text{-closed}$ in X . Hence $g \circ f$ is contra- $g\text{-irresolute}$.

Conclusion: In this paper we have introduced contra- $g\text{-irresolute}$ functions in topological spaces. Also, we have studied the relationship between contra- $g\text{-irresolute}$ functions and other continuous and irresolute functions already exist.

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