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**Abstract:** The aim of this paper is to introduce the notion of contra-generalized  $c^*$ -irresolute functions in topological spaces and study their basic properties. Also, we see that composition of two contra-generalized  $c^*$ -irresolute functions is contra-generalized  $c^*$ -irresolute function. This is the main part of this paper. Also, the contra-generalized  $c^*$ -irresolute function of a generalized  $c^*$ -irresolute function is contra-generalized  $c^*$ -irresolute function. Further, we prove contra-generalized  $c^*$ -irresolute function is the stronger form of contra- $gc^*$ -continuous function.

**Key words:**  $gc^*$ -continuous functions, contra- $gc^*$ -continuous functions,  $gc^*$ -irresolute functions and contra- $gc^*$ -irresolute functions.

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## 1. Introduction

In 1963, Norman Levine [5] introduced semi-open sets in topological spaces. Also in 1970, he introduced the concept of generalized closed sets. In 1980, Jain [4] introduced totally continuous functions. Dontchev [3] introduced the notions of contra continuity in topological spaces in 1996. In 2011, S.S. Benchalli and Umadevi I Neeli [1] introduced the concept of semi-totally continuous functions in topological spaces. In this paper we introduce contra-generalized  $c^*$ -irresolute functions in topological spaces and study their basic properties. Section 2 deals with the preliminary concepts. In section 3, contra-generalized  $c^*$ -irresolute functions are introduced and study their basic properties.

## 2. Preliminaries

Throughout this paper  $X$  denotes a topological space on which no separation axiom is assumed. For any subset  $A$  of  $X$ ,  $cl(A)$  denotes the closure of  $A$ ,  $int(A)$  denotes the interior of  $A$ . Further  $X \setminus A$  denotes the complement of  $A$  in  $X$ . The following definitions are very useful in the subsequent sections.

**Definition: 2.1** [5] A subset  $A$  of a topological space  $X$  is said to be a semi-open set if  $A \subseteq cl(int(A))$  and a semi-closed set if  $int(cl(A)) \subseteq A$ .

**Definition: 2.2** [14] A subset  $A$  of a topological space  $X$  is said to be a  $\alpha$ -open set if  $A \subseteq int(cl(int(A)))$  and a  $\alpha$ -closed set if  $cl(int(cl(A))) \subseteq A$ .

**Definition: 2.3** [6] A subset  $A$  of a topological space  $X$  is said to be a  $c^*$ -open set if  $int(cl(A)) \subseteq A \subseteq cl(int(A))$ .

**Definition: 2.4** [6] A subset  $A$  of a topological space  $X$  is said to be a generalized  $c^*$ -closed set (briefly,  $gc^*$ -closed set) if  $cl(A) \subseteq H$  whenever  $A \subseteq H$  and  $H$  is  $c^*$ -open. The complement of the  $gc^*$ -closed set is  $gc^*$ -open [7].

**Definition: 2.5** [9] A subset  $A$  of a topological space  $X$  is said to be a pre-generalized  $c^*$ -closed set (briefly,  $pgc^*$ -closed set) if  $pcl(A) \subseteq H$  whenever  $A \subseteq H$  and  $H$  is  $c^*$ -open. The complement of the  $pgc^*$ -closed set is  $pgc^*$ -open [10].

**Definition: 2.6** A function  $f: X \rightarrow Y$  is called

- i. totally-continuous [4] if the inverse image of every open subset of  $Y$  is clopen in  $X$ ,
- ii. strongly-continuous [15] if the inverse image of every subset of  $Y$  is clopen subset of  $X$ ,
- iii. semi-totally continuous [1] if the inverse image of every semi-open subset of  $Y$  is clopen in  $X$  &

iv. contra-continuous [3] if the inverse image of every open subset of  $Y$  is closed in  $X$ .

**Definition: 2.7** [8] A function  $f : X \rightarrow Y$  is called a generalized  $c^*$ -continuous (briefly,  $gc^*$ -continuous) function if the inverse image of every closed subset of  $Y$  is  $gc^*$ -closed in  $X$ .

**Definition: 2.8** [11] A function  $f : X \rightarrow Y$  is called a pre-generalized  $c^*$ -continuous (briefly,  $pgc^*$ -continuous) function if the inverse image of every closed subset of  $Y$  is  $pgc^*$ -closed in  $X$ .

**Definition: 2.9** [12] Let  $X$  and  $Y$  be two topological spaces. A function  $f : X \rightarrow Y$  is called a contra-generalized  $c^*$ -continuous (briefly, contra- $gc^*$ -continuous) function if  $f^{-1}(V)$  is  $gc^*$ -closed in  $X$  for every open set  $V$  of  $Y$ .

**Definition: 2.10** [13] Let  $X$  and  $Y$  be two topological spaces. A function  $f : X \rightarrow Y$  is called a contra-pre-generalized  $c^*$ -continuous (briefly, contra- $pgc^*$ -continuous) function if  $f^{-1}(V)$  is  $pgc^*$ -closed in  $X$  for every open set  $V$  of  $Y$ .

**Definition: 2.11** [2] A function  $f : X \rightarrow Y$  is called an irresolute function if the inverse image of every semi-open subset of  $Y$  is semi-open in  $X$ .

**Definition: 2.12** [8] A function  $f : X \rightarrow Y$  is called a generalized  $c^*$ -irresolute (briefly,  $gc^*$ -irresolute) function if the inverse image of every  $gc^*$ -closed subset of  $Y$  is  $gc^*$ -closed in  $X$ .

**Definition: 2.13** [11] A function  $f : X \rightarrow Y$  is called a pre-generalized  $c^*$ -irresolute (briefly,  $pgc^*$ -irresolute) function if the inverse image of every  $pgc^*$ -closed subset of  $Y$  is  $pgc^*$ -closed in  $X$ .

### 3. Contra-generalized $c^*$ -irresolute functions

In this section, we introduce contra-generalized  $c^*$ -irresolute functions and study their basic properties. Now, we begin with the definition of contra-generalized  $c^*$ -irresolute function.

**Definition: 3.1** Let  $X$  and  $Y$  be two topological spaces. A function  $f : X \rightarrow Y$  is said to be a contra-generalized  $c^*$ -irresolute (briefly, contra- $gc^*$ -irresolute) function if  $f^{-1}(V)$  is  $gc^*$ -closed in  $X$  for every  $gc^*$ -open set  $V$  of  $Y$ .

**Example: 3.2** Let  $X = \{1, 2, 3, 4\}$  and  $Y = \{a, b, c, d\}$ . Then, clearly  $\tau = \{\emptyset, \{1\}, \{2, 3, 4\}, X\}$  is a topology on  $X$  and  $\sigma = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}, Y\}$  is a topology on  $Y$ . Define  $f$  by  $f(1)=b$ ,  $f(2)=d$ ,  $f(3)=c$ ,  $f(4)=a$ . Then the inverse image of every  $gc^*$ -open set in  $Y$  is  $gc^*$ -closed in  $X$ . Therefore,  $f$  is contra- $gc^*$ -irresolute.

**Proposition: 3.3** Let  $X, Y$  be two topological spaces. Then  $f: X \rightarrow Y$  is contra- $gc^*$ -irresolute iff  $f^{-1}(U)$  is  $gc^*$ -open in  $X$  for every  $gc^*$ -closed set  $U$  of  $Y$ .

**Proof:** Assume that  $f : X \rightarrow Y$  is contra- $gc^*$ -irresolute. Let  $U$  be a  $gc^*$ -closed set in  $Y$ . Then  $Y \setminus U$  is a  $gc^*$ -open set in  $Y$ . This implies,  $f^{-1}(Y \setminus U)$  is a  $gc^*$ -closed set in  $X$ . Since  $f^{-1}(Y \setminus U) = X \setminus f^{-1}(U)$ , we have  $X \setminus f^{-1}(U)$  is a  $gc^*$ -closed set in  $X$ . This implies,  $f^{-1}(U)$  is a  $gc^*$ -open set in  $X$ . Conversely, assume that  $f^{-1}(U)$  is  $gc^*$ -open in  $X$  for every  $gc^*$ -closed set  $U$  in  $Y$ . Let  $V$  be a  $gc^*$ -open set in  $Y$ . Then  $Y \setminus V$  is  $gc^*$ -closed in  $Y$ . Therefore,  $f^{-1}(Y \setminus V)$  is  $gc^*$ -open in  $X$ . That is,  $X \setminus f^{-1}(V)$  is  $gc^*$ -open in  $X$ . This implies,  $f^{-1}(V)$  is  $gc^*$ -closed in  $X$ . Therefore,  $f$  is contra- $gc^*$ -irresolute.

**Proposition: 3.4** Let  $X, Y$  be two topological spaces. Then every contra- $gc^*$ -irresolute function is contra- $gc^*$ -continuous.

**Proof:** The proof follows easily from the fact that every closed set is  $gc^*$ -closed.

The following example shows that the converse of Proposition 3.4 need not be true.

**Example: 3.5** Let  $X = \{a, b, c, d\}$  with topology  $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}, X\}$  and  $Y = \{1, 2, 3, 4\}$  with topology  $\sigma = \{\emptyset, \{1\}, \{3\}, \{1, 3\}, \{2, 3, 4\}, Y\}$ . Define  $f: X \rightarrow Y$  by  $f(a)=4$ ,  $f(b)=2$ ,  $f(c)=1$ ,  $f(d)=3$ . Then the inverse image of every open set in  $Y$  is  $gc^*$ -closed in  $X$ . Therefore,  $f$  is contra- $gc^*$ -continuous. But  $f$  is not a contra- $gc^*$ -irresolute, since the inverse image of the  $gc^*$ -open set  $\{2\}$  is  $\{b\}$ , which is not a  $gc^*$ -closed set in  $X$ .

**Proposition: 3.6** Let  $X, Y$  be two topological spaces. Then every contra- $gc^*$ -irresolute function is contra- $pgc^*$ -continuous.

**Proof:** Let  $f : X \rightarrow Y$  be a contra-gc\*-irresolute function. Let  $V$  be a closed set in  $Y$ . Then  $V$  is gc\*-closed in  $Y$ . By our assumption,  $f^{-1}(V)$  is gc\*-open in  $X$ . This implies,  $f^{-1}(V)$  is pgc\*-open in  $X$ . Therefore,  $f$  is contra-pgc\*-continuous.

The converse of Proposition 3.6 need not be true as seen from the following example.

**Example: 3.7** Let  $X = \{a, b, c, d\}$  with topology  $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, b, c\}, X\}$  and  $Y = \{1, 2, 3, 4, 5\}$  with topology  $\sigma = \{\emptyset, \{1, 2\}, \{3, 4\}, \{1, 2, 3, 4\}, Y\}$ . Define  $f: X \rightarrow Y$  by  $f(a)=2, f(b)=1, f(c)=4, f(d)=3$ . Then the inverse image of every open set in  $Y$  is pgc\*-closed in  $X$ . Therefore,  $f$  is contra-pgc\*-continuous. But  $f$  is not a contra-gc\*-irresolute, since the inverse image of the gc\*-open set  $\{1\}$  in  $Y$  is  $\{b\}$ , which is not a gc\*-closed set in  $X$ .

**Proposition: 3.8** Let  $X, Y$  and  $Z$  be topological spaces. Then

1. If  $f : X \rightarrow Y$  is contra-gc\*-irresolute and  $g : Y \rightarrow Z$  is continuous (resp. gc\*-continuous), then  $g \circ f : X \rightarrow Z$  is contra-gc\*-continuous,
2. If  $f : X \rightarrow Y$  is contra-gc\*-irresolute and  $g : Y \rightarrow Z$  is gc\*-irresolute, then  $g \circ f : X \rightarrow Z$  is contra-gc\*-irresolute,
3. If  $f : X \rightarrow Y$  is contra-gc\*-irresolute and  $g : Y \rightarrow Z$  is gc\*-irresolute, then  $g \circ f : X \rightarrow Z$  is contra-gc\*-continuous &
4. If  $f : X \rightarrow Y$  is gc\*-irresolute and  $g : Y \rightarrow Z$  is contra-gc\*-irresolute, then  $g \circ f : X \rightarrow Z$  is contra-gc\*-irresolute.

**Proof:** It is obvious.

The following Proposition shows that the composition of contra-gc\*-irresolute function and some functions is contra-pgc\*-continuous function.

**Proposition: 3.9** Let  $X, Y$  and  $Z$  be topological spaces. Then

1. If  $f : X \rightarrow Y$  is contra-gc\*-irresolute and  $g : Y \rightarrow Z$  is continuous, then  $g \circ f : X \rightarrow Z$  is contra-pgc\*-continuous,
2. If  $f : X \rightarrow Y$  is contra-gc\*-irresolute and  $g : Y \rightarrow Z$  is gc\*-continuous, then  $g \circ f : X \rightarrow Z$  is contra-pgc\*-continuous,
3. If  $f : X \rightarrow Y$  is pgc\*-irresolute and  $g : Y \rightarrow Z$  is contra-gc\*-irresolute, then  $g \circ f : X \rightarrow Z$  is contra-pgc\*-continuous.

**Proposition: 3.10** Let  $X, Y$  and  $Z$  be topological spaces.

1. If  $f : X \rightarrow Y$  is contra-gc\*-irresolute and  $g : Y \rightarrow Z$  is contra-continuous (resp. contra-gc\*-continuous), then  $g \circ f : X \rightarrow Z$  is gc\*-continuous,
2. If  $f : X \rightarrow Y$  is contra-gc\*-irresolute and  $g : Y \rightarrow Z$  is contra-gc\*-continuous, then  $g \circ f : X \rightarrow Z$  is gc\*-continuous (resp. pgc\*-continuous),
3. If  $f : X \rightarrow Y$  and  $g : Y \rightarrow Z$  are contra-gc\*-irresolute, then  $g \circ f : X \rightarrow Z$  is gc\*-continuous (resp. pgc\*-continuous) &
4. If  $f : X \rightarrow Y$  and  $g : Y \rightarrow Z$  are contra-gc\*-irresolute, then  $g \circ f : X \rightarrow Z$  is gc\*-irresolute.

**Proof:** The proof is trivial.

**Proposition: 3.11** Let  $X, Y$  and  $Z$  be topological spaces. Then

1. If  $f : X \rightarrow Y$  is contra-gc\*-irresolute and  $g : Y \rightarrow Z$  is totally-continuous, then  $g \circ f : X \rightarrow Z$  is gc\*-continuous (resp. pgc\*-continuous),
2. If  $f : X \rightarrow Y$  is contra-gc\*-irresolute and  $g : Y \rightarrow Z$  is strongly-continuous, then  $g \circ f : X \rightarrow Z$  is gc\*-continuous (resp. pgc\*-continuous),
3. If  $f : X \rightarrow Y$  is contra-gc\*-irresolute and  $g : Y \rightarrow Z$  is semi-totally continuous, then  $g \circ f : X \rightarrow Z$  is gc\*-continuous (resp. pgc\*-continuous).

**Proposition: 3.12** Let  $X, Y$  and  $Z$  be topological spaces. Then

1. If  $f : X \rightarrow Y$  is contra-gc\*-irresolute and  $g : Y \rightarrow Z$  is totally-continuous, then  $g \circ f : X \rightarrow Z$  is contra-gc\*-continuous,

2. If  $f: X \rightarrow Y$  is contra-gc\*-irresolute and  $g: Y \rightarrow Z$  is strongly-continuous, then  $g \circ f: X \rightarrow Z$  is contra-gc\*-continuous,
3. If  $f: X \rightarrow Y$  is contra-gc\*-irresolute and  $g: Y \rightarrow Z$  is semi-totally continuous, then  $g \circ f: X \rightarrow Z$  is contra-gc\*-continuous.

In other words, totally-continuous (resp. strongly-continuous, semi-totally continuous) function of a contra-gc\*-irresolute function is contra-gc\*-continuous.

**Proposition: 3.13** Let  $X, Y$  and  $Z$  be topological spaces. Then

1. If  $f: X \rightarrow Y$  is contra-gc\*-irresolute and  $g: Y \rightarrow Z$  is totally-continuous, then  $g \circ f: X \rightarrow Z$  is contra-pgc\*-continuous,
2. If  $f: X \rightarrow Y$  is contra-gc\*-irresolute and  $g: Y \rightarrow Z$  is strongly-continuous, then  $g \circ f: X \rightarrow Z$  is contra-pgc\*-continuous &
3. If  $f: X \rightarrow Y$  is contra-gc\*-irresolute and  $g: Y \rightarrow Z$  is semi-totally continuous, then  $g \circ f: X \rightarrow Z$  is contra-pgc\*-continuous.

**Proof:** The proof follows from the fact that every gc\*-closed set is pgc\*-closed.

Irresolute and contra-gc\*-irresolute functions are independent of each other. For example, let  $X = \{1, 2, 3, 4\}$  and  $Y = \{a, b, c, d\}$ . Then, clearly  $\tau = \{\emptyset, \{1\}, \{3\}, \{4\}, \{1, 3\}, \{1, 4\}, \{3, 4\}, \{1, 3, 4\}, X\}$  is a topology on  $X$  and  $\sigma = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}, Y\}$  is a topology on  $Y$ . Define  $f: X \rightarrow Y$  by  $f(1)=b, f(2)=d, f(3)=c, f(4)=a$ . Then  $f$  is irresolute. But  $f$  is not contra-gc\*-irresolute, since the inverse image of the gc\*-open set  $\{a, c\}$  is  $\{3, 4\}$ , which is not a gc\*-closed set in  $X$ . Define  $g: X \rightarrow Y$  by  $g(1)=g(3)=g(4)=a, g(2)=b$ . Then  $g$  is contra-gc\*-irresolute. But the inverse image of the semi-open set  $\{b, c, d\}$  is  $\{2\}$ , which is not a semi-open set in  $X$ . Therefore,  $g$  is not irresolute.

**Proposition: 3.14** Let  $X, Y$  and  $Z$  be topological spaces. If  $f: X \rightarrow Y$  and  $g: Y \rightarrow Z$  are contra-gc\*-irresolute, then  $g \circ f: X \rightarrow Z$  is contra-gc\*-irresolute.

**Proof:** Let  $X = \{x_1, x_2, \dots, x_n\}$ ,  $Y = \{y_1, y_2, \dots, y_m\}$  and  $Z = \{z_1, z_2, \dots, z_p\}$  with  $n < m < p$ . Then there are four cases arise.

**Case (i) :** Suppose  $f$  and  $g$  are 1-1. Define  $f \& g$  by  $f(x_i) = y_i$  for all  $i = 1, 2, \dots, n$  and  $g(y_j) = z_j$  for all  $j = 1, 2, \dots, m$ . Since each  $\{y_i\}$  is gc\*-open and  $f$  is contra-gc\*-irresolute, we have  $\{f^{-1}(y_i)\} = \{x_i\}$  is gc\*-closed. That is, each  $\{x_i\}$  is gc\*-closed in  $X$ . Since union of gc\*-closed sets is gc\*-closed, we have every subset of  $X$  is gc\*-closed in  $X$ . This implies, inverse image of every gc\*-open set in  $Z$  under  $g \circ f$  is gc\*-closed in  $X$ . Therefore,  $g \circ f$  is contra-gc\*-irresolute.

**Case (ii) :** Suppose  $f$  is 1-1 and  $g$  is not 1-1. Define  $f$  by  $f(x_1) = y_1, f(x_2) = y_2, \dots, f(x_n) = y_n$  and define  $g$  by  $g(y_1) = g(y_2) = \dots = g(y_k) = z_k, g(y_{k+1}) = z_{k+1}, \dots, g(y_m) = z_m$ , where  $1 < k < m$ . Since every singleton sets are gc\*-open, we have  $\{z_k\}, \{z_{k+1}\}, \dots, \{z_m\}$  are gc\*-open in  $Z$  &  $\{y_1\}, \{y_2\}, \dots, \{y_n\}$  are gc\*-open in  $Y$ . Also, since  $f$  and  $g$  are contra-gc\*-irresolute, we have  $g^{-1}(\{z_k\}), g^{-1}(\{z_{k+1}\}), \dots, g^{-1}(\{z_n\})$  are gc\*-closed in  $Y$  and  $f^{-1}(\{y_1\}), f^{-1}(\{y_2\}), \dots, f^{-1}(\{y_n\})$  are gc\*-closed in  $X$ . That is,  $\{y_1, y_2, \dots, y_k\}, \{y_{k+1}\}, \dots, \{y_m\}$  are gc\*-closed in  $Y$  and  $\{x_1\}, \{x_2\}, \dots, \{x_n\}$  are gc\*-closed in  $X$ . Since union of gc\*-closed sets in  $X$  is gc\*-closed in  $X$ , we have every subset of  $X$  is gc\*-closed in  $X$ . This implies, inverse image of every gc\*-open set in  $Z$  under  $g \circ f$  is gc\*-closed in  $X$ . Therefore,  $g \circ f$  is contra-gc\*-irresolute.

**Case (iii) :** Suppose  $g$  is 1-1 and  $f$  is not 1-1. Define  $f$  by  $f(x_1) = f(x_2) = \dots = f(x_k) = y_k, f(x_{k+1}) = y_{k+1}, \dots, f(x_n) = y_n$ , where  $1 < k < n$  and define  $g$  by  $g(y_1) = z_1, g(y_2) = z_2, \dots, g(y_m) = z_m$ . Since each  $\{z_i\}$  and each  $\{y_i\}$  are gc\*-open in  $Z$  and  $Y$  respectively and  $f \& g$  are contra-gc\*-irresolute, we have each  $\{y_i\}$  is gc\*-closed in  $Y$  and  $\{x_1, x_2, \dots, x_k\}, \{x_{k+1}\}, \dots, \{x_n\}$  are gc\*-closed in  $X$ . Now,

$$(g \circ f)^{-1}(\{z_1\}) = f^{-1}(g^{-1}(\{z_1\})) = f^{-1}(\{y_1\}) = \emptyset,$$

$$(g \circ f)^{-1}(\{z_2\}) = f^{-1}(g^{-1}(\{z_2\})) = f^{-1}(\{y_2\}) = \emptyset,$$

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$$\begin{aligned}
(g \circ f)^{-1}(\{z_{k-1}\}) &= f^{-1}(g^{-1}(\{z_{k-1}\})) = f^{-1}(\{y_{k-1}\}) = \phi, \\
(g \circ f)^{-1}(\{z_k\}) &= f^{-1}(g^{-1}(\{z_k\})) = f^{-1}(\{y_k\}) = \{x_1, x_2, \dots, x_k\}, \\
(g \circ f)^{-1}(\{z_{k+1}\}) &= f^{-1}(g^{-1}(\{z_{k+1}\})) = f^{-1}(\{y_{k+1}\}) = \{x_{k+1}\}, \\
&\vdots \\
&\vdots \\
&\vdots
\end{aligned}$$

$$\begin{aligned}
(g \circ f)^{-1}(\{z_n\}) &= f^{-1}(g^{-1}(\{z_n\})) = f^{-1}(\{y_n\}) = \{x_n\}, \\
(g \circ f)^{-1}(\{z_{n+1}\}) &= f^{-1}(g^{-1}(\{z_{n+1}\})) = f^{-1}(\{y_{n+1}\}) = \phi, \\
&\vdots \\
&\vdots \\
&\vdots
\end{aligned}$$

$$\begin{aligned}
(g \circ f)^{-1}(\{z_m\}) &= f^{-1}(g^{-1}(\{z_m\})) = f^{-1}(\{y_m\}) = \phi, \\
(g \circ f)^{-1}(\{z_{m+1}\}) &= f^{-1}(g^{-1}(\{z_{m+1}\})) = f^{-1}(\{\phi\}) = \phi, \\
&\vdots \\
&\vdots \\
&\vdots
\end{aligned}$$

$$(g \circ f)^{-1}(\{z_p\}) = f^{-1}(g^{-1}(\{z_p\})) = f^{-1}(\{\phi\}) = \phi, \text{ which are } gc^*-\text{closed in } X.$$

That is, each  $(g \circ f)^{-1}(\{z_i\})$  is  $gc^*$ -closed in  $X$ .

If  $\{z_1, z_2, \dots, z_s\}$ ,  $1 \leq s \leq p$  is  $gc^*$ -open in  $Z$ , then  $(g \circ f)^{-1}(\{z_1, z_2, \dots, z_s\}) = (g \circ f)^{-1}(\{z_1\}) \cup (g \circ f)^{-1}(\{z_2\}) \cup \dots \cup (g \circ f)^{-1}(\{z_s\})$ .

Since the union of  $gc^*$ -closed sets in  $gc^*$ -closed, we have  $(g \circ f)^{-1}(\{z_1, z_2, \dots, z_s\})$  is  $gc^*$ -closed. Therefore, inverse image of every  $gc^*$ -open set in  $Z$  under  $g \circ f$  is  $gc^*$ -closed in  $X$ . Hence  $g \circ f$  is contra- $gc^*$ -irresolute.

**Case (iv) :** Suppose  $f$  and  $g$  are not 1-1. Define  $f$  by  $f(x_1) = f(x_2) = \dots = f(x_k) = y_k$ ,  $f(x_{k+1}) = y_{k+1}, \dots, f(x_n) = y_n$ , where  $1 < k < n$ . Define  $g$  by  $g(y_1) = g(y_2) = \dots = g(y_r) = z_r$ ,  $g(y_{r+1}) = z_{r+1}, \dots, g(y_m) = z_m$ , where  $1 < r < m$ . Since each  $\{z_i\}$  and each  $\{y_i\}$  are  $gc^*$ -open in  $Z$  and  $Y$  respectively and  $f, g$  are contra- $gc^*$ -irresolute, we have  $\{x_1, x_2, \dots, x_k\}, \{x_{k+1}\}, \dots, \{x_n\}$  are  $gc^*$ -closed in  $X$  and  $\{y_1, y_2, \dots, y_r\}, \{y_{r+1}\}, \dots, \{y_m\}$  are  $gc^*$ -closed in  $Y$ . Now,

$$\begin{aligned}
(g \circ f)^{-1}(\{z_1\}) &= f^{-1}(g^{-1}(\{z_1\})) = f^{-1}(\{\phi\}) = \phi, \\
&\vdots \\
&\vdots \\
&\vdots \\
(g \circ f)^{-1}(\{z_{r-1}\}) &= f^{-1}(g^{-1}(\{z_{r-1}\})) = f^{-1}(\{\phi\}) = \phi, \\
(g \circ f)^{-1}(\{z_r\}) &= f^{-1}(g^{-1}(\{z_r\})) = f^{-1}(\{y_1, y_2, \dots, y_r\}) \\
&= f^{-1}(\{y_1\}) \cup f^{-1}(\{y_2\}) \cup \dots \cup f^{-1}(\{y_r\}) \\
&= \phi \cup \phi \cup \dots \cup f^{-1}(\{y_r\}) \\
&= f^{-1}(\{y_r\}) \\
&= \begin{cases} \phi & \text{if } r < k \\ \{x_1, x_2, \dots, x_k\} \cup \{x_{k+1}\} \cup \dots \cup \{x_r\} \cup \{x_{r+1}\} & \text{if } r \geq k \end{cases} \\
&= \begin{cases} \phi & \text{if } r < k \\ \{x_1, x_2, \dots, x_{r+1}\} & \text{if } r \geq k \end{cases} \\
(g \circ f)^{-1}(\{z_{r+1}\}) &= f^{-1}(g^{-1}(\{z_{r+1}\})) = f^{-1}(\{y_{r+1}\}) \\
&= \begin{cases} \phi & \text{if } r+1 < k \\ \{x_1, x_2, \dots, x_k\} \cup \{x_{k+1}\} \cup \dots \cup \{x_r\} \cup \{x_{r+1}\} & \text{if } r+1 \geq k \end{cases} \\
&= \begin{cases} \phi & \text{if } r < k-1 \\ \{x_1, x_2, \dots, x_{r+1}\} & \text{if } r \geq k-1 \end{cases}, \\
&\vdots \\
&\vdots \\
&\vdots
\end{aligned}$$

$$(g \circ f)^{-1}(\{z_n\}) = f^{-1}(g^{-1}(\{z_n\})) = f^{-1}(\{y_n\}) = \{x_n\},$$

$$(g \circ f)^{-1}(\{z_{n+1}\}) = f^{-1}(g^{-1}(\{z_{n+1}\})) = f^{-1}(\{y_{n+1}\}) = \phi,$$

$$(g \circ f)^{-1}(\{z_m\}) = f^{-1}(g^{-1}(\{z_m\})) = f^{-1}(\{y_m\}) = \phi,$$

$$(g \circ f)^{-1}(\{z_{m+1}\}) = f^{-1}(g^{-1}(\{z_{m+1}\})) = f^{-1}(\{\phi\}) = \phi,$$

$$(g \circ f)^{-1}(\{z_p\}) = f^{-1}(g^{-1}(\{z_p\})) = f^{-1}(\{\phi\}) = \phi, \text{ which are } gc^*-\text{closed in } X.$$

That is, each  $(g \circ f)^{-1}(\{z_i\})$  is  $gc^*$ -closed in  $X$ .

If  $\{z_1, z_2, \dots, z_q\}$ ,  $1 < q < p$  is  $gc^*$ -open in  $Z$ , then  $(g \circ f)^{-1}(\{z_1, z_2, \dots, z_q\}) = (g \circ f)^{-1}(\{z_1\}) \cup (g \circ f)^{-1}(\{z_2\}) \cup \dots \cup (g \circ f)^{-1}(\{z_q\})$ .

Since the union of  $gc^*$ -closed sets in  $X$  is  $gc^*$ -closed, we have  $(g \circ f)^{-1}(\{z_1, z_2, \dots, z_s\})$  is  $gc^*$ -closed. Therefore, inverse image of every  $gc^*$ -open set in  $Z$  under  $g \circ f$  is  $gc^*$ -closed in  $X$ . Hence  $g \circ f$  is contra- $gc^*$ -irresolute.

**Conclusion:** In this paper we have introduced contra- $gc^*$ -irresolute functions in topological spaces. Also, we have studied the relationship between contra- $gc^*$ -irresolute functions and other continuous and irresolute functions already exist.

## References

- [1] S.S. Benchalli and U. I Neeli, Semi-totally Continuous function in topological spaces, Inter. Math. Forum , 6 (2011) ,10,479-492.
- [2] S.G. Crossley and S.K. Hildebrand, Semi-topological properties, Fund. Math.,74 (1972), 233-254.
- [3] J. Dontchev, Contra-continuous functions and strongly S-closed spaces, Int. J. Math. & Math. Sci.,19(1996), 303-310.
- [4] R.C. Jain, The role of regularly open sets in general topological spaces, Ph.D.,thesis, Meerut University, Institute of advanced studies, Meerut-India, (1980).
- [5] N. Levine, Semi-open sets and semi-continuity in topological space, Amer. Math. Monthly., 70 (1963), 39-41.
- [6] S. Malathi and S. Nithyanantha Jothi, On  $c^*$ -open sets and generalized  $c^*$ -closed sets in topological spaces, Acta ciencia indica, Vol. XLIII M, No.2, 125 (2017), 125-133.
- [7] S. Malathi and S. Nithyanantha Jothi, On generalized  $c^*$ -open sets and generalized  $c^*$ -open maps in topological spaces, Int. J. Mathematics And its Applications, Vol. 5, issue 4-B (2017), 121-127.
- [8] S. Malathi and S. Nithyanantha Jothi, "On generalized  $c^*$ -continuous functions and generalized  $c^*$ -irresolute functions in topological spaces, Turkish Journal of Analysis and Number Theory, Vol. 6, no.6, (2018), 164-168.
- [9] S. Malathi and S. Nithyanantha Jothi, On Pre-generalized  $c^*$ -closed sets in topological spaces, Journal of Computer and Mathematical Sciences Vol. 8 (12) (2017), 720-726.
- [10] S. Malathi and S. Nithyanantha Jothi, On Pre-generalized  $c^*$ -open sets and Pre-generalized  $c^*$ -open maps in topological spaces, Int. J. Mathematical Archive, Vol. 8 (12) (2017), 66-70.
- [11] S. Malathi and S. Nithyanantha Jothi, On Pre-generalized  $c^*$ -continuous functions and Pre-generalized  $c^*$ -irresolute functions in topological spaces, Mathematical Sciences International

Research Journal, Vol. 7, Spl issue 4, (2018), 17-22.

- [12] S. Malathi and S. Nithyanantha Jothi, "On Contra-generalized  $c^*$ -continuous functions in topological spaces", Proceedings of National Seminar on New Dimensions in Mathematics and its Applications, (2019), 30-37.
- [13] S. Malathi and S. Nithyanantha Jothi, "On Contra Pre-generalized  $c^*$ -continuous functions in topological spaces", Emerging Trends in pure and Applied Mathematics (Conference Proceedings), (2019), 53-59.
- [14] O. Njastad, On some classes of nearly open sets, Pacific J. Math., 15 (1965), 961-970.
- [15] M. Stone, Application of the theory of Boolean rings to general topology, Trans. Amer. Math. Soc., 41(1937), 374-481.